

Holographic Superconductor/Insulator Transition with logarithmic electromagnetic field in Gauss-Bonnet gravity

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Abstract

The behaviors of the holographic superconductors/insulator transition are studied by introducing a charged scalar field coupled with a logarithmic electromagnetic field in both the Einstein-Gauss-Bonnet AdS black hole and soliton. For the Einstein-Gauss-Bonnet AdS black hole, we find that: i) the larger coupling parameter of logarithmic electrodynamic field b makes it easier for the scalar hair to be condensated; ii) the ratio of the gap frequency in conductivity ω_g to the critical temperature T_c depends on both b and the Gauss-Bonnet constant α ; and iii) the critical exponents are independent of the b and α . For the Einstein-Gauss-Bonnet AdS Soliton, we show that the system is the insulator phase when the chemical potential μ is small, but there is a phase transition and the AdS soliton reaches the superconductor (or superfluid) phase when μ larger than critical chemical potential. A special property should be noted is that the critical chemical potential is not changed by the coupling parameter b but depends on α .

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I. INTRODUCTION

The anti-de Sitter/conformal field theory (AdS/CFT) correspondence [1–3] indicates that a weak coupling gravity theory in a d -dimensional anti-de Sitter spacetime can be related to a strong coupling conformal field theory on the $(d - 1)$ -dimensional boundary. The AdS/CFT duality is a powerful tool for investigating strongly coupled gauge theories, the application might offer new insight into the study of strongly interacting condensed matter systems where the perturbational methods are no longer available. Therefore, much attention has been given to the studies of the AdS/CFT duality to condensed matter physics and in particular to superconductivity recently [4–22]. In these studies most of the holographically dual descriptions for a superconductor are based on a model that a simple Einstein-Maxwell theory coupled to a charged scalar.

Since Heisenberg and Euler [23] noted that quantum electrodynamics predicts that the electromagnetic field behaves nonlinearly through the presence of virtual charged particles, the nonlinear electrodynamics has been an interesting subject for many years [24–32] because the nonlinear electrodynamics carries more information than the Maxwell field. One of the important nonlinear electrodynamics is the logarithmic electromagnetic field which appears in the description of vacuum polarization effects. The logarithmic terms were obtained as exact 1-loop corrections for electrons in a uniform electromagnetic field background by Euler and Heisenberg [23]. Ref. [33] presented a Born-Infeld-like Lagrangian with a logarithmic term which can be added as a correction to the original Born-Infeld one. The logarithmic electromagnetic lagrangian takes the form $\mathcal{L}_{BI} = -b^2 \ln(1 + F^2/b^2)$ where b is a coupling constant. The Lagrangian tends to the Maxwell case in the weak-coupling limit $b \rightarrow \infty$.

Within the framework of AdS/CFT correspondence, the properties of holographic superconductors for a given black hole or soliton depend on behavior of the nonlinear electromagnetic field coupled with the charged scalar field. We [34] investigated the behaviors of the holographic superconductors by introducing a charged scalar field coupled with a Power-Maxwell field and found that the larger power parameter makes it harder for the scalar hair to be condensated. We [35] also studied the holographic superconductors in Gauss-Bonnet gravity with Born-Infeld electrodynamics and noted that the model parameters and the Born-Infeld coupling parameter will affect the formation of the scalar hair, the transition point of the phase transition from the second order to the first order, and the relation connecting the gap frequency in conductivity with the critical temperature. In this paper we will study the behaviors of the holographic superconductors/insulator

transition by introducing a charged scalar field coupled with a logarithmic electromagnetic field in both the Einstein-Gauss-Bonnet AdS black hole and soliton, and to see how the logarithmic electromagnetic field affect the formation of the scalar hair, the critical exponent, and the critical chemical potential of the systems.

The paper is organized as follows. In Sec. II, we give the holographic dual of the Einstein-Gauss-Bonnet AdS black hole by introducing a charged scalar field coupled with a logarithmic electromagnetic field. In Sec. III, the behaviors of the holographic superconductors/insulator transition are studied by coupling a charged scalar field with a logarithmic electromagnetic field in the Einstein-Gauss-Bonnet AdS soliton. We summarize and discuss our conclusions in the last section.

II. EINSTEIN-GAUSS-BONNET ADS₅ BLACK HOLE AND SUPERCONDUCTOR

The action of the Einstein-Gauss-Bonnet theory, which is the most general Lovelock theory in five and six dimensions, is given by

$$I_{grav} = \frac{1}{16\pi G} \int_{\mathcal{M}} d^d x \sqrt{-g} \left[R - 2\Lambda + \hat{\alpha} \left(R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\lambda\sigma}R^{\mu\nu\lambda\sigma} \right) \right], \quad (2.1)$$

where $\Lambda = -(d-1)(d-2)/(2L^2)$ is the cosmological constant, and $\hat{\alpha}$ is the Gauss-Bonnet coupling constant. The static spacetime in the Einstein-Gauss-Bonnet gravity is described by [36, 37]

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 dx_i dx^i, \quad (2.2)$$

with

$$f(r) = \frac{r^2}{2\alpha} \left[1 - \sqrt{1 - \frac{4\alpha}{L^2} \left(1 - \frac{ML^2}{r^{d-1}} \right)} \right], \quad (2.3)$$

where the constant M is relate to the black hole horizon by $r_+ = (ML^2)^{1/(d-1)}$ and $\alpha = \hat{\alpha}(d-3)(d-4)$. We can define the effective asymptotic AdS scale as $L_{\text{eff}}^2 = 2\alpha/(1 - \sqrt{1 - \frac{4\alpha}{L^2}})$ because $f(r) \sim \frac{r^2}{2\alpha} \left(1 - \sqrt{1 - 4\alpha/L^2} \right)$ in the asymptotic region ($r \rightarrow \infty$). The Hawking temperature of the Einstein-Gauss-Bonnet AdS black hole, which will be interpreted as the temperature of the CFT, can be expressed as

$$T = \frac{(d-1)r_+}{4\pi L^2}. \quad (2.4)$$

In the background of the d -dimensional Einstein-Gauss-Bonnet AdS black hole, we consider the logarithmic electrodynamic field and the charged scalar field coupled via a generalized action

$$S = \int d^d x \sqrt{-g} \left[-2b^2 \ln \left(1 + \frac{F_{\mu\nu} F^{\mu\nu}}{8b^2} \right) - \frac{1}{2} \partial_\mu \tilde{\psi} \partial^\mu \tilde{\psi} - \frac{1}{2} m^2 \tilde{\psi}^2 - \frac{\tilde{\psi}^2}{2} (\partial_\mu p - A_\mu)(\partial^\mu p - A^\mu) \right], \quad (2.5)$$

where $F^{\mu\nu}$ is the strength of the logarithmic electrodynamic field $F = dA$, $\tilde{\psi}$ is a scalar field, and b is a coupling constant. The logarithmic electrodynamic field will reduce to the Maxwell case as $b \rightarrow \infty$. We can use the gauge freedom to fix $p = 0$ and take $\psi \equiv \tilde{\psi}$ and $A_t = \phi$, where both ψ and ϕ are real functions of r only. Then we can obtain the following equations of motion

$$\psi'' + \left(\frac{f'}{f} + \frac{d-2}{r} \right) \psi' + \frac{\phi^2}{f^2} \psi - \frac{m^2}{f} \psi = 0, \quad (2.6)$$

$$\phi'' \left(1 + \frac{\phi'^2}{4b^2} \right) + \frac{d-2}{r} \left(1 - \frac{\phi'^2}{4b^2} \right) \phi' - \frac{2\psi^2 \phi}{f} \left(1 - \frac{\phi'^2}{4b^2} \right)^2 = 0, \quad (2.7)$$

where a prime denotes the derivative with respect to r . At the event horizon $r = r_+$ of the black hole, we must have

$$\begin{aligned} \phi(r_+) &= 0, \\ \psi(r_+) &= -\frac{(d-1)r_+}{m^2 L^2} \psi'(r_+), \end{aligned} \quad (2.8)$$

and at the asymptotic AdS region ($r \rightarrow \infty$), the solutions behave like

$$\psi = \frac{\psi_-}{r^{\lambda_-}} + \frac{\psi_+}{r^{\lambda_+}}, \quad \phi = \mu - \frac{\rho}{r^{d-3}}, \quad (2.9)$$

with

$$\lambda_{\pm} = \frac{1}{2} \left[(d-1) \pm \sqrt{(d-1)^2 + 4m^2 L_{eff}^2} \right], \quad (2.10)$$

where μ and ρ are interpreted as the chemical potential and charge density in the dual field theory, respectively. The coefficients ψ_+ and ψ_- both multiply normalizable modes of the scalar field equations and they correspond to the vacuum expectation values $\psi_+ = \langle O_+ \rangle$, $\psi_- = \langle O_- \rangle$ of an operator O dual to the scalar field according to the AdS/CFT correspondence. We can impose boundary conditions that either ψ_+ or ψ_- vanishes in the following study.

A. Relations between critical temperature and charge density

In this subsection we will present a detail analysis of the condensation of the operator $\langle O_+ \rangle$ and $\langle O_- \rangle$ by taking numerical integration of the equations (2.6) and (2.7) from the event horizon out to the infinity with the boundary conditions mentioned above.

The influence of the parameters b and α on the condensation with fixed values $m^2 L_{eff}^2 = -3$ is presented in Fig. 1, in which two panels in the left column for the operator $\langle O_+ \rangle$ and panels in the right column are for the operator $\langle O_- \rangle$. We know from the figure that the value of the operator $\langle O_+ \rangle$ decreases as the coupling parameter b of the logarithmic electrodynamics increases with a fixed Gauss-Bonnet parameter α , but the value of the operator $\langle O_- \rangle$ increases as b increases for the same α . It should point out that the curves for both $b = 100$ and $b = 1000$ almost overlap.

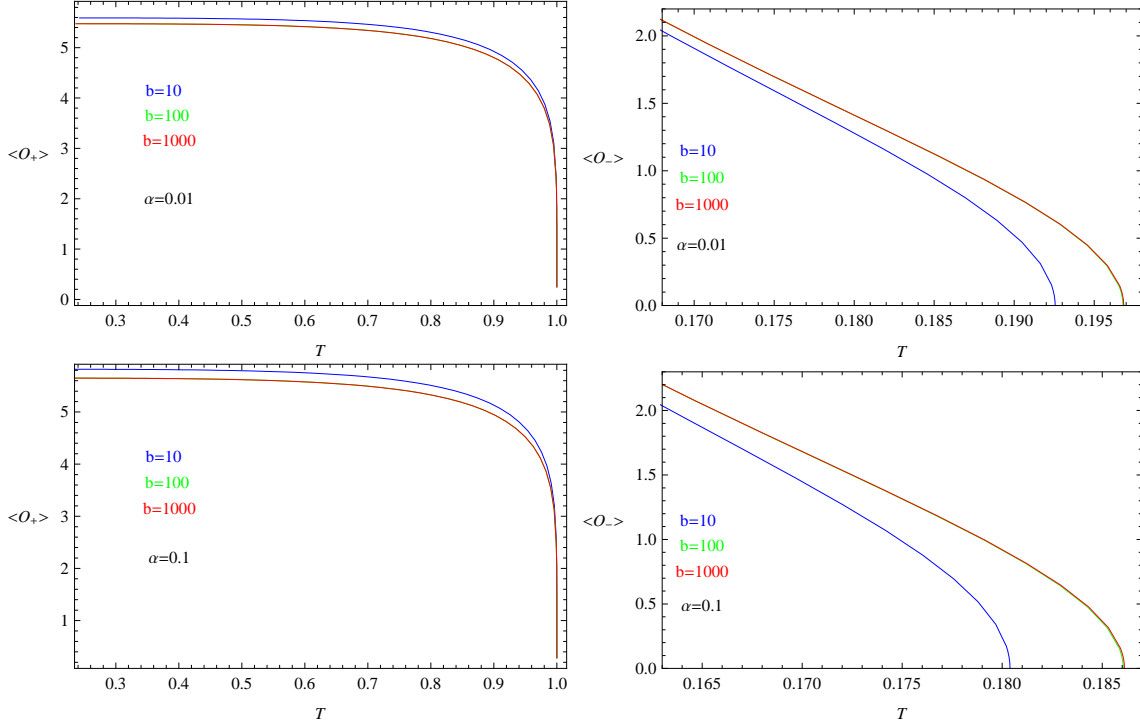


FIG. 1: (Color online.) The condensate for operators $\langle O_+ \rangle$ (left column) and $\langle O_- \rangle$ (right column) as a function of the temperature for different values of the coupling parameter b and Gauss-Bonnet parameter α . In each panel the lines for 100 (green) and 1000 (red) are almost overlap.

In Table I, we list the values of the critical temperature for different values of b and α in the 5-dimensional black hole, respectively. For both the scalar operators $\langle O_+ \rangle$ and $\langle O_- \rangle$, we find from the table I that the critical temperature increases as the value of b increases with fixed α , which means that the larger parameter b makes it easier for the scalar hair to be condensated in 5-dimensional Einstein-Gauss-Bonnet AdS black hole; however, the critical temperature decreases as α increases for the fixed b , which means that the stronger Gauss-Bonnet coupling makes condensate harder in the black hole.

TABLE I: The critical values of T_c for different b and α in the 5-dimensional Einstein-Gauss-Bonnet AdS black hole with $m^2 L_{eff}^2 = -3.75$.

	$b = 1000$		$b = 100$		$b = 10$	
	T_c for $\langle O_+ \rangle$	T_c for $\langle O_- \rangle$	T_c for $\langle O_+ \rangle$	T_c for $\langle O_- \rangle$	T_c for $\langle O_+ \rangle$	T_c for $\langle O_- \rangle$
$\alpha = 0.00$	$0.219643\rho^{1/3}$	$0.312928\rho^{1/3}$	$0.219618\rho^{1/3}$	$0.312925\rho^{1/3}$	$0.217230\rho^{1/3}$	$0.312580\rho^{1/3}$
$\alpha = 0.01$	$0.219004\rho^{1/3}$	$0.312078\rho^{1/3}$	$0.218979\rho^{1/3}$	$0.312074\rho^{1/3}$	$0.216555\rho^{1/3}$	$0.311725\rho^{1/3}$
$\alpha = 0.05$	$0.216291\rho^{1/3}$	$0.308494\rho^{1/3}$	$0.216264\rho^{1/3}$	$0.308491\rho^{1/3}$	$0.213676\rho^{1/3}$	$0.308119\rho^{1/3}$
$\alpha = 0.10$	$0.212456\rho^{1/3}$	$0.303548\rho^{1/3}$	$0.212436\rho^{1/3}$	$0.303543\rho^{1/3}$	$0.209578\rho^{1/3}$	$0.303139\rho^{1/3}$

B. Critical exponents

In this subsection, we will study the critical exponents of the holographic superconductor model with the logarithmic electromagnetic field by using numerical method. In Fig. 2, we present the condensate of the operators $\langle O_+ \rangle$ (left column) and $\langle O_- \rangle$ (right column) as a function of $(1 - T/T_c)$ in logarithmic scale with different values of b and α for $d = 5$. The top two panels for $\alpha = 0.01$ and bottom two for $\alpha = 0.1$, and in each panel the three lines correspond to $b = 10, 100$ and 1000 which are almost overlap. We see from these panels that the slope is independent of the parameters b and α , which is in agreement with the value $1/2$. It should be pointed out that the result seems to be a universal property for various nonlinear electrodynamics if the scalar field ψ takes the form of this paper.

C. Electrical Conductivity

Horowitz *et al.* [5] got following relation connecting the gap frequency in conductivity with the critical temperature for (2+1) and (3+1)-dimensional superconductors

$$\frac{\omega_g}{T_c} \approx 8, \quad (2.11)$$

which is roughly twice the BCS value 3.5 indicating that the holographic superconductors are strongly coupled. We now examine this relation for the Gauss-Bonnet gravity with the logarithmic electrodynamic field.

In the computation of the electrical conductivity, the perturbation of the logarithmic electromagnetic field should be considered in the Gauss-Bonnet black hole background. However, in the

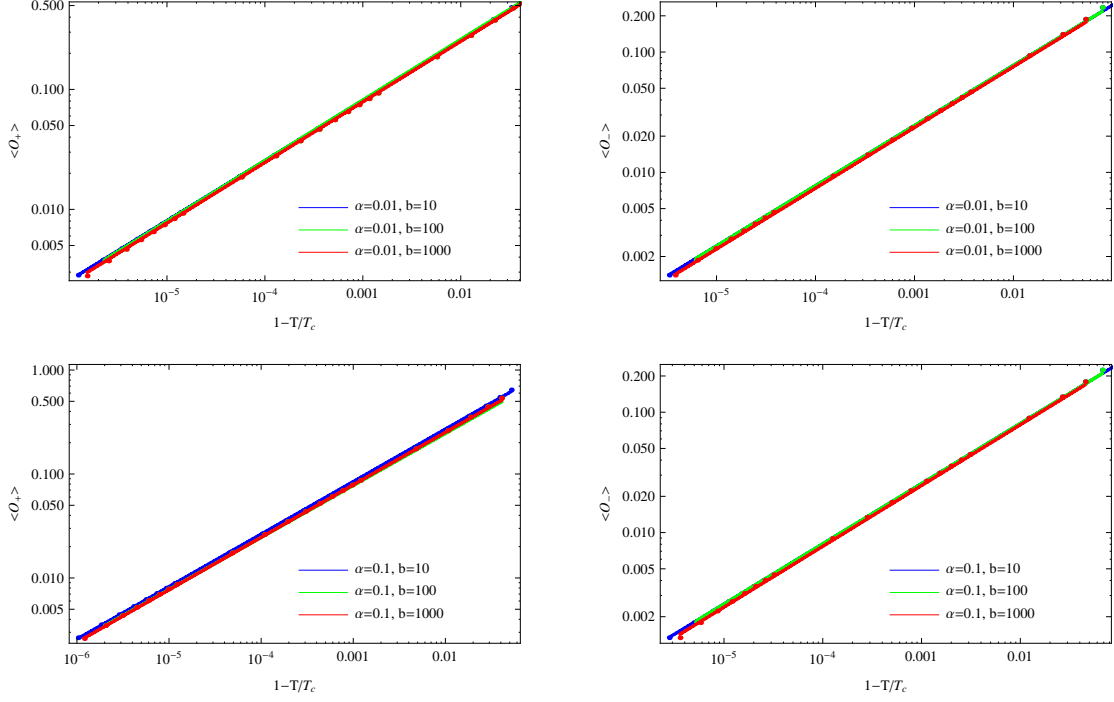


FIG. 2: (Color online.) The critical exponents for operators $\langle O_+ \rangle$ (left column) and $\langle O_- \rangle$ (right column) as a function of the temperature for different values of the coupling parameter b and Gauss-Bonnet parameter α . In each panel the three lines for $b = 10$ (blue), 100 (green) and 1000 (red) are almost overlap.

probe approximation we can ignore the effect of the perturbation of the metric. Assuming that the perturbation is translational symmetric and has a time dependence as $\delta A_x = A_x(r)e^{-i\omega t}$, we find that the motion equation for the logarithmic electrodynamic field in the Gauss-Bonnet black hole background can be written as

$$\left[A_x'' + \left(\frac{f'}{f} + \frac{d-4}{r} \right) A_x' + \frac{\omega^2}{f^2} A_x \right] \left(1 - \frac{\phi'^2}{4b^2} \right) + \frac{\phi'\phi''}{2b^2} A_x - \left(1 - \frac{\phi'^2}{4b^2} \right)^2 \frac{2\psi^2}{f} A_x = 0. \quad (2.12)$$

Noting that an ingoing wave boundary condition near the horizon is $A_x(r) \sim f(r)^{-\frac{i\omega L^2}{(d-1)r_+}}$, and a general behavior for $d = 5$ in the asymptotic AdS region ($r \rightarrow \infty$) [20] is $A_x = L_e^{-1/2} A^{(0)} + \frac{L_e^{3/2}}{r^2} \left(A^{(2)} - \frac{1}{2} \ln \frac{r}{L} \partial_t^2 A^{(0)} \right)$, we find that the holographic conductivity can be expressed as [20]

$$\sigma = \frac{2A^{(2)}}{i\omega A^{(0)}} + \frac{i\omega}{2} - i\omega \log \frac{L_e}{L}, \quad (2.13)$$

Solving the motion equation (2.12) numerically we can obtain the conductivity.

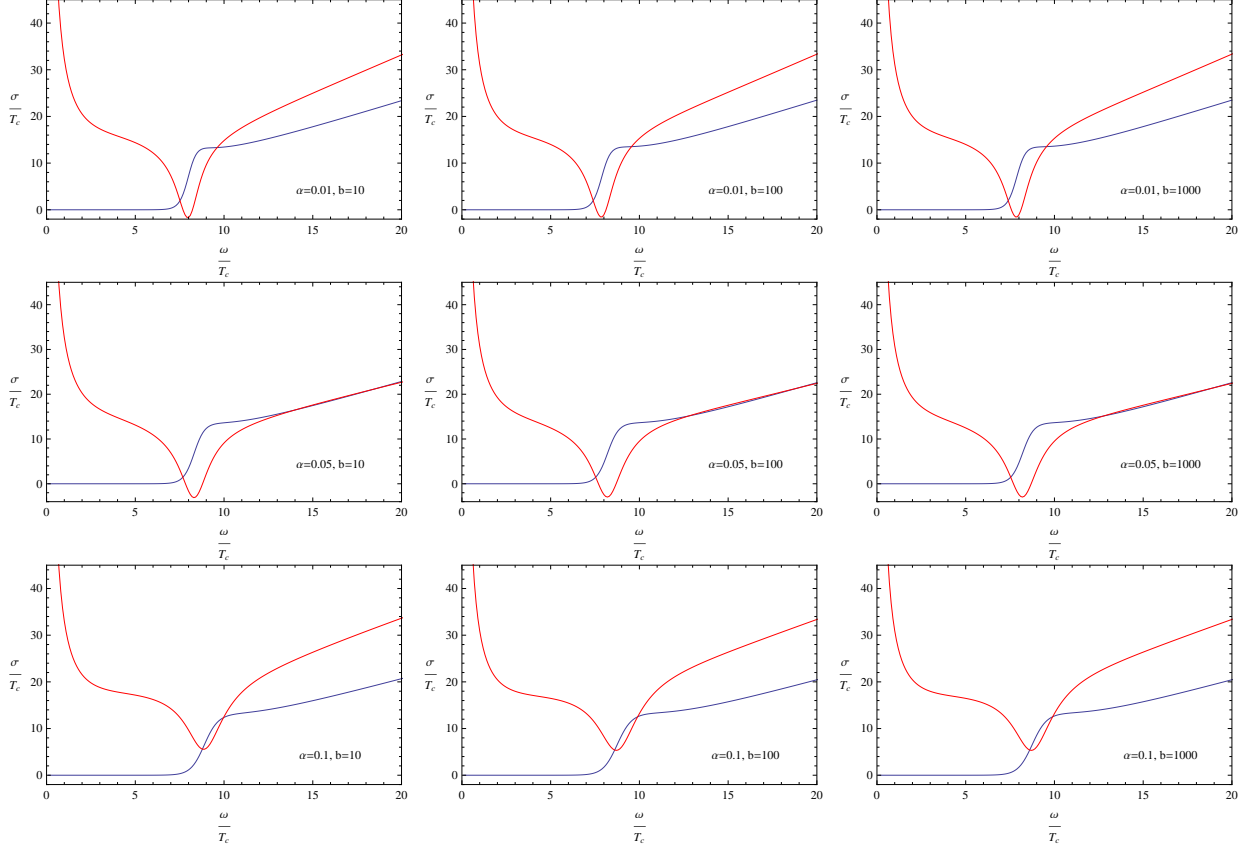


FIG. 3: (Color online) The conductivity of the superconductors as a function of ω/T_c for different values of b and α with fixed $m^2 L_{eff}^2 = -3$. In each panel the blue (bottom) line represents the real part of the conductivity, $Re(\sigma)$, and red (top) line is the imaginary part of the conductivity, $Im(\sigma)$.

TABLE II: The ratio ω_g/T_c for different values of the Gauss-Bonnet constant α and the coupling parameter b with $m^2 L_{eff}^2 = -3$.

	b=1000	b=100	b=10
$\alpha = 0.01$	7.7	7.8	7.9
$\alpha = 0.05$	8.0	8.1	8.2
$\alpha = 0.10$	8.7	8.8	8.9

We present in Fig. 3 and table II the frequency dependent conductivity obtained by solving the motion equation of the logarithmic electrodynamic field numerically for different values of α and b with $m^2 L_{eff}^2 = -3$ (we plot the conductivity at temperature $T/T_c \simeq 0.25$). We note that the gap frequency ω_g decreases with the increase of the coupling parameter b for fixed α , it increases as α increases for fixed b . From Fig. 3 and table II, we find that the ratio of the gap frequency in

conductivity ω_g to the critical temperature T_c in the Gauss-Bonnet black hole with the logarithmic electrodynamic field depends on both the Gauss-Bonnet constant and the coupling parameter of logarithmic electrodynamic field.

III. EINSTEIN-GAUSS-BONNET ADS SOLITON AND HOLOGRAPHIC SUPERCONDUCTOR

In this section we will study a holographic superconductor for a logarithmic electrodynamic field coupled with a charged scalar field in a Einstein-Gauss-Bonnet AdS soliton. The metric of the Einstein-Gauss-Bonnet AdS soliton is described by [38]

$$ds^2 = -r^2 dt^2 + \frac{dr^2}{f(r)} + f(r) d\varphi^2 + r^2 h_{ij} dx^i dx^j, \quad (3.1)$$

where $f(r) = \frac{r^2}{2\alpha} \left[1 - \sqrt{1 - \frac{4\alpha}{L^2} \left(1 - \frac{r_s^{d-1}}{r^{d-1}} \right)} \right]$. This spacetime does not have any horizon but a conical singularity at $r = r_s$ which can be removed by imposing a period $\beta = \frac{4\pi L^2}{(d-1)r_s}$ for the coordinate φ .

Using the generalized action (2.5) for the logarithmic electrodynamic field coupled with the charged scalar field, we find that the equations of motion are given by

$$\psi'' + \left(\frac{f'}{f} + \frac{d-2}{r} \right) \psi' + \left(\frac{\phi^2}{r^2 f} - \frac{m^2}{f} \right) \psi = 0, \quad (3.2)$$

$$\left(1 + \frac{f\phi'^2}{4b^2 r^2} \right) \phi'' + \left(\frac{f'}{f} + \frac{d-4}{r} \right) \phi' - \frac{(d-2)f}{4b^2 r^3} \phi'^3 - \left(1 - \frac{f\phi'^2}{4b^2 r^2} \right) \frac{2\psi^2}{f} \phi = 0. \quad (3.3)$$

The asymptotic solutions of the functions ψ and ϕ near the AdS boundary ($r \rightarrow \infty$) are the same as Eq. (2.9), but at the tip $r = r_s$ of the soliton, the solutions must satisfy

$$\begin{aligned} \psi &= \tilde{\psi}_0 + \tilde{\psi}_1 \log(r - r_s) + \tilde{\psi}_2 (r - r_s) + \dots, \\ \phi &= \tilde{\phi}_0 + \tilde{\phi}_1 \log(r - r_s) + \tilde{\phi}_2 (r - r_s) + \dots, \end{aligned} \quad (3.4)$$

where $\tilde{\psi}_i$ and $\tilde{\phi}_i$ ($i = 0, 1, 2, \dots$) are the integration constants. We should impose the Neumann-like boundary condition $\tilde{\psi}_1 = \tilde{\phi}_1 = 0$ [39] to keep every physical quantity finite. The probe approximation will also be used in our discussion.

In Figs. 4 and 5 we plot the condensations of scalar operators $\langle O_+ \rangle_S$ (left column) and $\langle O_- \rangle_S$ (right column) with respect to the chemical potential μ in the Einstein-Gauss-Bonnet AdS Soliton for different logarithmic electrodynamics parameter b and Gauss-Bonnet coupling constant α with the fixed scalar mass $m^2 L_{\text{eff}}^2 = -3.75$. The condensation occurs for scalar operators $\langle O_i \rangle_S$ ($i = \pm$) with different values of b and α if $\mu > \mu_{iS}$, where μ_{iS} is the so-called critical chemical potential for

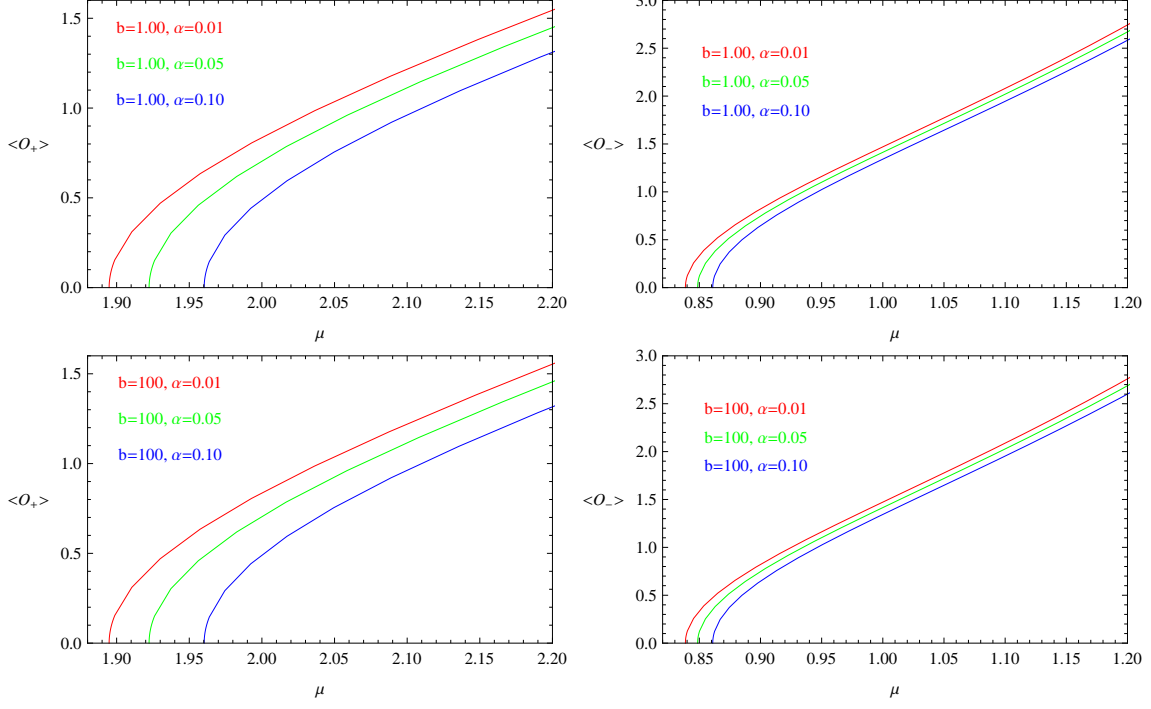


FIG. 4: (Color online) The condensates of the scalar operators $\langle O_+ \rangle_S$ (left column) and $\langle O_- \rangle_S$ (right column) with respect to the chemical potential μ in the Gauss-Bonnet Soliton. From top row to bottom one we take $b = 1$ and $b = 100$ with fixed $m^2 L_{\text{eff}}^2 = -3.75$. In each panel, the three lines from left to right correspond to increasing α , i.e., $\alpha = 0.01$ (red), 0.05 (green) and 0.1 (blue) respectively.

TABLE III: The critical chemical potential μ_{+S} and μ_{-S} for the scalar operators $\langle O_+ \rangle_S$ and $\langle O_- \rangle_S$ in the 5-dimensional Einstein-Gauss-Bonnet AdS soliton with $m^2 L_{\text{eff}}^2 = -3.75$.

	$b = 1000$		$b = 100$		$b = 10$	
	μ_{+S}	μ_{-S}	μ_{+S}	μ_{-S}	μ_{+S}	μ_{-S}
$\alpha = 0.00$	1.88832	0.83618	1.88832	0.83618	1.88832	0.83618
$\alpha = 0.01$	1.89484	0.83863	1.89484	0.83863	1.89484	0.83863
$\alpha = 0.05$	1.92234	0.84860	1.92234	0.84860	1.92234	0.84860
$\alpha = 0.10$	1.96025	0.86098	1.96025	0.86098	1.96025	0.86098

scalar operators $\langle O_i \rangle_S$ which just begin to condense. The μ_{+S} and μ_{-S} for scalar operators $\langle O_+ \rangle_S$ and $\langle O_- \rangle_S$ with different values of b and α are listed in table III. From the figures and table we find that μ_{iS} increase as α increases with fixed b , while μ_{iS} does not change for different b with fixed α .

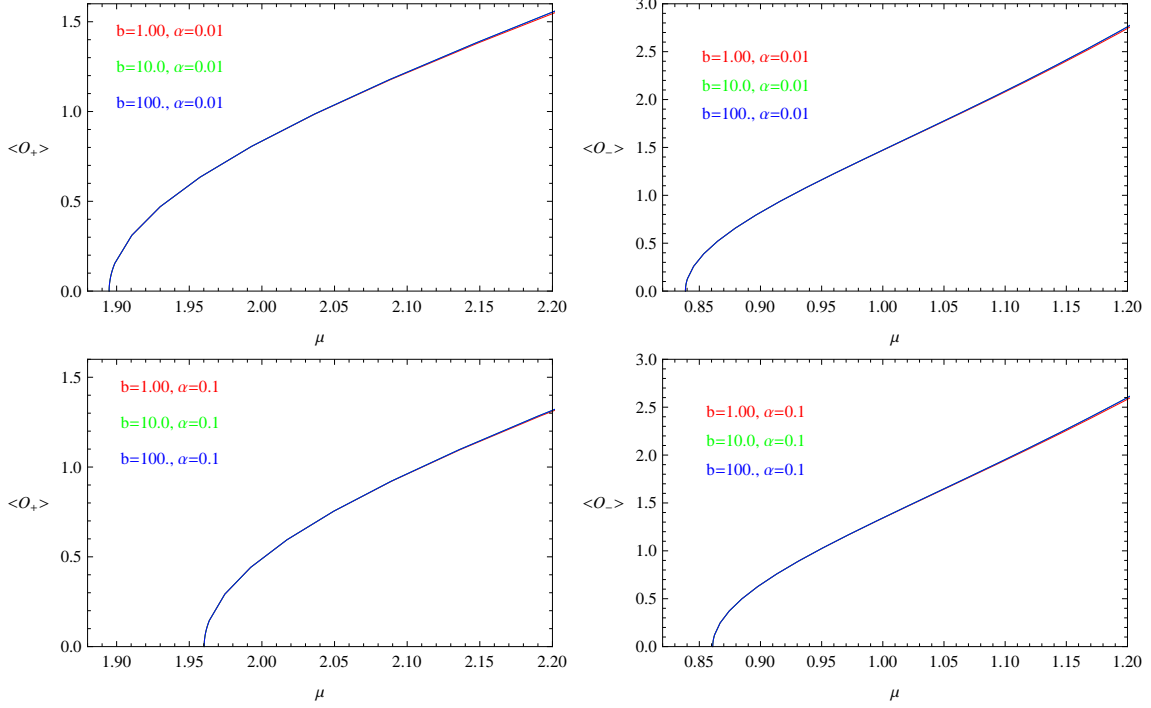


FIG. 5: (Color online) The condensates of the scalar operators $\langle O_+ \rangle_S$ (left column) and $\langle O_- \rangle_S$ (right column) with respect to the chemical potential μ in the Gauss-Bonnet Soliton. From top row to bottom one we take $\alpha = 0.01$ and $\alpha = 0.1$ with fixed $m^2 L_{\text{eff}}^2 = -3.75$. In each panel the three lines for $b = 1$ (red), 10 (green) and 100 (blue) are almost overlap.

Figs. 6 and 7 show that the charge density ρ as a function of the chemical potential μ when $\langle O_+ \rangle \neq 0$ (left) and $\langle O_- \rangle \neq 0$ (right). We see from these figures that, when μ is small, the system is described by the AdS soliton solution itself which is the insulator phase [39], however, when μ reaches μ_{+S} or μ_{-S} , there is a phase transition and the AdS soliton reaches the superconductor (or superfluid) phase. Fig. 6 shows that the phase transition will begin later as α increases for fixed b , while Fig. 7 tells us that the phase transition will take place at the same point for different b if we fixed α .

IV. CONCLUSIONS

The behaviors of the holographic superconductors/insulator transition have been investigated by introducing a charged scalar field coupled with a logarithmic electromagnetic field in both the Einstein-Gauss-Bonnet AdS black hole and soliton.

For the Einstein-Gauss-Bonnet AdS black hole, we find that the critical temperature increases

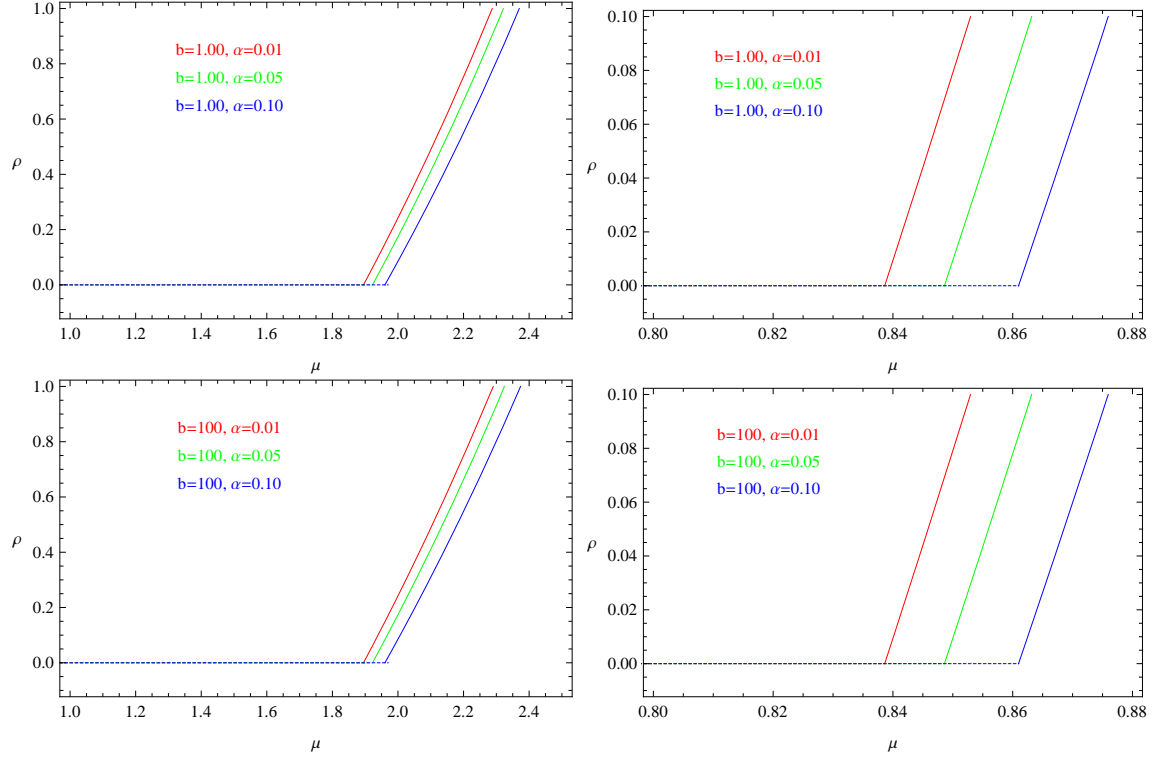


FIG. 6: (color online) The charge density ρ as a function of the chemical potential μ with different values of b and α when $\langle O_+ \rangle_S \neq 0$ (left) and $\langle O_- \rangle_S \neq 0$ (right). From top row to bottom one we take $b = 1$ and $b = 100$ with fixed $m^2 L_{\text{eff}}^2 = -15/4$. The three lines from left to right correspond to increasing α , i.e., 0.01 (red), 0.05 (green) and 0.1 (blue) respectively.

as the value of b increases with fixed α , which means that the larger parameter b makes it easier for the scalar hair to be condensate in the Einstein-Gauss-Bonnet AdS black hole; however, the critical temperature decreases as α increases for the fixed b , which means that the stronger Gauss-Bonnet coupling makes condensated harder in the black hole. We note that the gap frequency ω_g decreases with the increase of the coupling parameter b for fixed α , but it increases as α increases for fixed b . The ratio of the gap frequency in conductivity ω_g to the critical temperature T_c in the Einstein-Gauss-Bonnet AdS black hole with the logarithmic electrodynamic field depends on both the Gauss-Bonnet constant and the coupling parameter of logarithmic electrodynamic field. We also show that the critical exponent is independent of the parameters b and α , which is in agreement with the value $1/2$. The result seems to be a universal property for various nonlinear electrodynamics if the scalar field ψ takes the form of this paper.

For the Einstein-Gauss-Bonnet AdS Soliton, the system is the insulator phase when μ is small,

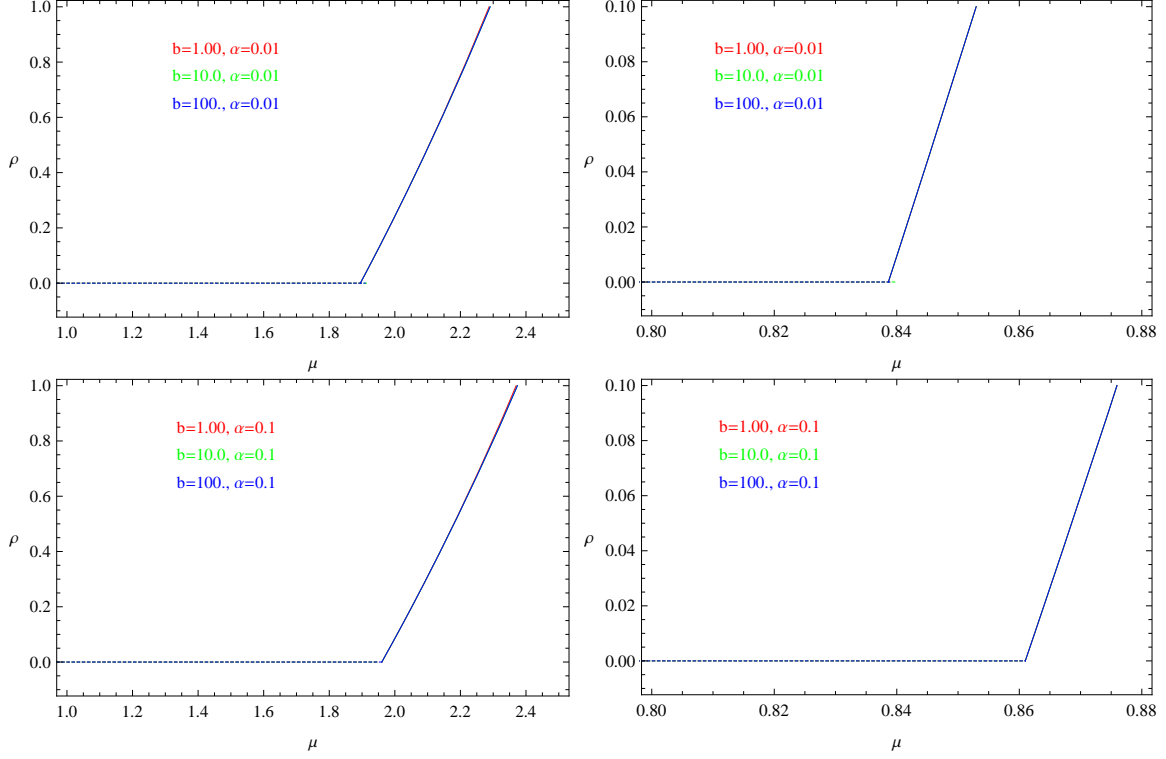


FIG. 7: (color online) The charge density ρ as a function of the chemical potential μ with different values of α when $\langle O_+ \rangle_S \neq 0$ (left) and $\langle O_- \rangle_S \neq 0$ (right) with fixed mass $m^2 L_{\text{eff}}^2 = -3.75$. From top row to bottom one we take $\alpha = 0.01$ and $\alpha = 0.1$. In each panel the three lines for $b = 1$ (red), 10 (green) and 100 (blue) are almost overlap.

but there is a phase transition and the AdS soliton reaches the superconductor (or superfluid) phase when μ larger than the critical chemical potential μ_{+S} or μ_{-S} . Especially, the phase transition can occur even at strictly zero temperature. We note that the critical chemical potential μ_{iS} ($i = \pm 1$) increases as α increases with fixed b , while it does not change by the coupling constant b with fixed α . That is to say, the phase transition will begin later as α increases for fixed b , while the phase transition will take place at the same point for different b with fixed α .

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Key Discipline.

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